Non-Linear Bias Mitigation in Multi-Sensor Multi-Track Fusion

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***Abstract*—When performing track correlation and fusion in conjunction with bias estimation for sensor registration, the pattern match bias estimation is usually performed by modeling the biases as additive constants to the tracks in Cartesian space. Since sensor biases actually occur in sensor polar or spherical coordinates, the bias model of adding constants to the tracks can only be applied to a group of somewhat closely-spaced tracks before the linear assumption of the biases in Cartesian coordinates breaks down. A methodology to estimate sensor biases in the native coordinate frame in which they occur is presented, along with simulation results that illustrate its performance. Modeling the biases in sensor coordinates allow for tracks throughout the field of view to be used for sensor bias estimation, producing better sensor registration and track picture. In this research, sensor tracks are transmitted to a fusion center, where track correlation, bias estimation, and fusion are performed. Murty's K-best hypotheses algorithm is utilized to generated the top K hypotheses for track-to-track correlation. Each hypothesis produces an estimate of the sensor biases. The correlation hypotheses are corrected for their sensor bias estimates and new correlation scores are computed, and the bias-corrected correlation hypotheses are ranked to find the best. The best hypothesis is selected as the most recent system track picture. The system tracks produced by the best hypothesis are correlated against the previous system track picture to maintain system track number continuity. The performance of the bias estimation is assessed against the root mean squared error (RMSE) and normalized estimation error squared (NEES) errors of the estimated biases versus the true biases. A scenario with four tracks and two sensors is used to demonstrate the observability of these biases. The results show that the biases as applied to the remote sensor are observable and mitigated, allowing for a more accurate track picture.**

I. Introduction

Modern tracking systems are trending towards the use case where a fused system track picture at the highest level is desired to be able to manage a single integrated picture of the environment with multiple sensors at independent sites. Each of these sites have systems designed to maintain their independent track picture with tracks representing their view of the environment. However, when attempting to fuse these tracks together with multiple, independent sites, a major obstacle to successful fusion is the presence of biases in the track data between each platform.

In the multi-sensor, multi-platform environment, biases can take many different forms. Some of the major sources of these biases are due to imperfect sensor calibration and sensor registration errors in the location of the sensor. While methods currently exist for mitigating the effects of these sources of error, residual biases can remain. These residual biases can seem minor on a sensor per sensor basis, but when multiple sensors are introduced, this results in a difficult environment to perform track correlation and track fusion without further corrupting the track picture. One approach to mitigating this problem is inflating track covariances to account for the unmitigated biases, but this may be insufficient because covariance inflation for a fixed error is a modeling mismatch. Inflation of the track covariance to account for residual sensor biases tends to distort the underlying pattern or relative locations of closely spaced targets, and that distortion will hamper any further downstream correlation of those system tracks to other source tracks [7].

When observing tracks in a full 360 degree picture, calculating the biases via a linear approximation of the sensor biases, as presented in [5], is insufficient to mitigate them. In [5], the targets were assumed to be sufficiently close enough in angle that the biases can be assumed to be constant in Cartesian space, where the track state is defined. For sensors that have biases that cover a large angle space, this assumption is inappropriate and the biases must be observed in their native coordinate frame in which the measurements are being produced.

To estimate the biases on sensor coordinates, a sufficient number of common, well-distributed tracks are required to make the biases observable. However, prior to estimation of the sensor biases, the tracks from the multiple sensors must be properly correlated. A global nearest neighbor (GNN) assignment algorithm is required to take advantage of the patterns in the overall picture. Attempting track correlation in the presence of sensor biases with a standard nearest neighbor (SNN) assignment algorithm will fail, and inaccurate bias estimates will result.

In this research, sensor tracks are transmitted to a fusion center, where track correlation, bias estimation, and fusion are performed. Murty's K-best hypotheses algorithm is employed to generated the top K hypotheses for track-to-track correlation [2]. Each hypothesis produces an estimate of the sensor biases. The differences of the correlated tracks are written directly as the function of the sensor biases rather than the more complex (although likely more precise) formulation found in [8], [9]. The correlation hypotheses are corrected for their sensor bias estimates and new correlation scores are computed, and the bias-corrected correlation hypotheses are ranked to find the best. The best hypothesis is selected as the most recent system track picture. The system tracks produced by the best hypothesis are correlated against the previous system track picture to maintain system track number continuity. The performance of the bias estimation is assessed against the root mean squared error (RMSE) in biases and normalized estimation error squared (NEES). A scenario with four tracks and two sensors is used to demonstrate the observability of these biases. The results show that the biases as applied to the remote sensor are observable and mitigated, allowing for a more accurate track picture.

This paper is organized as follows. Section II defines the notation and provides background material. Section III formulates the observations of the sensor biases and develops the nonlinear least squares estimation algorithm for the sensor biases. Correlation and fusion of the multi-sensor tracks are addressed in Section IV. Section V provides simulation results. Conclusions are offered in Section VI.

II. Background

Let X_k denote the kinematic state vector of the target at time t_k. It typically contains the position, velocity, and possibly acceleration of the target, as well as other variables used to model a time-varying acceleration. The kinematic model commonly assumed for a maneuvering target in track [1] is given by

$$X_{k+1} = F_k X_k + G_k v_k, \tag{1}$$

where F_k defines the linear constraint on the target kinematics between times k and $k+1$, and $v_k \sim N(0, Q_k)$ is the white random process that accounts for uncertainty in the linear dynamics. The model for radar measurements is a nonlinear model of the target state in Cartesian coordinates given by

$$Z_k = h(X_k) + w_k, \tag{2}$$

where Z_k is the typical radar measurement of range and angles for the target and $w_k \sim N(0, R_k)$ are the white noise observation errors. Both w_k and v_k are assumed to be independent white Gaussian error processes. For this paper, the radar measures range and azimuth. The covariance R_k will includes variances of the measurements in range $\sigma^2_{r|k}$ and azimuth $\sigma^2_{\theta|k}$.

Let there be N targets that are observed by L sensors. The observations of target j from sensor i at time t_k are modeled in radar reference coordinates as

$$\begin{aligned} Z_k^{j|i} &= \begin{bmatrix} r_k^{j|i} \\ \theta_k^{j|i} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{(x_k^{j|i})^2 + (y_k^{j|i})^2} \\ \tan^{-1}(\frac{y_k^{j|i}}{x_k^{j|i}}) \end{bmatrix} + \begin{bmatrix} \delta_r^i \\ \delta_\theta^i \end{bmatrix} + \begin{bmatrix} w_{r,k}^{j|i} \\ w_{\theta,k}^{j|i} \end{bmatrix} \\ &= h(X_k^{j|i}) + \delta^i + \begin{bmatrix} w_{r,k}^{j|i} \\ w_{\theta,k}^{j|i} \end{bmatrix}, \end{aligned} \tag{3}$$

where $(x_k^{j|i})$ and $(y_k^{j|i})$ are position of Target j in Sensor i Cartesian reference coordinates and δ_r^i and δ_θ^i are the biases in Sensor i. For the Extended Kalman Filter (EKF),

$$H_k^{j|i} = \left.\frac{\partial h(X_k)}{\partial X_k}\right|_{X_k = X_{k|k-1}^{j|i}}, \tag{4}$$

where $X_k = X_{k|k-1}^{j|i}$ is the predicted target position in the coordinates of the Sensor i.

III. Bias Estimation

To estimate the sensor biases, the track state estimates must be written in terms of the sensor biases. Writing the target state in terms of the sensor biases gives

$$\begin{aligned} X_k^i &= \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} = g(Z_k^i, \delta^i) \\ &= \begin{bmatrix} (r_k^i - \delta_r^i)\cos(\theta_k^i - \delta_\theta) \\ r_k^i \dot{r}_k^i \cos(\theta_k^i - \delta_\theta^i) - (r_k^i - \delta_r^i)\dot{\theta}_k^i \sin(\theta_k^i - \delta_\theta^i) \\ (r_k^i - \delta_r^i)\sin(\theta_k^i - \delta_\theta) \\ r_k^i \dot{r}_k^i \sin(\theta_k^i - \delta_\theta^i) - (r_k^i - \delta_r^i)\dot{\theta}_k^i \cos(\theta_k^i - \delta_\theta^i) \end{bmatrix}. \end{aligned} \tag{5}$$

This shows that the impact of the biases on the track state is clearly nonlinear. A nonlinear least squares estimation approach will be taken to the estimate the biases and an iterative numerical procedure will be required. Let the n^{th} iteration of the bias estimate be denoted as

$$B_n = \begin{bmatrix} \delta_n^1 \\ \delta_n^2 \end{bmatrix} = \begin{bmatrix} \delta_{r_n}^1 \\ \delta_{\theta_n}^1 \\ \delta_{r_n}^2 \\ \delta_{\theta_n}^2 \end{bmatrix}. \tag{6}$$

Observations of the bias can be formed by taking the difference of the state estimates of the various sensors tracking the same target. For N tracks from two distributed sensors, the general observation equation is given by

$$\begin{aligned} Y_k = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} &= \begin{bmatrix} X_{k|k}^{1|1} - M^{21}X^{1(1)|2} + T^{21} \\ X_{k|k}^{2|2} - M^{21}X^{1(2)|2} + T^{21} \\ \vdots \\ X_{k|k}^{N|N} - M^{21}X^{1(N)|2} + T^{21} \end{bmatrix} = \begin{bmatrix} g(Z_{k|k}^{1|1}, \delta^1) - M^{21}g(Z^{1(1)|2}, \delta^1) + T^{21} \\ g(Z_{k|k}^{2|1}, \delta^1) - M^{21}g(Z^{1(2)|2}, \delta^1) + T^{21} \\ \vdots \\ g(Z_{k|k}^{N|1}, \delta^1) - M^{21}g(Z^{1(N)|2}, \delta^1) + T^{21} \end{bmatrix} + \begin{bmatrix} W_k^{1|1} - M^{21}W_k^{1(1)|2} \\ W_k^{2|1} - M^{21}W_k^{1(2)|2} \\ \vdots \\ W_k^{N|1} - M^{21}W_k^{1(N)|2} \end{bmatrix} \\ &= \bar{g}(\{Z_k^{j|1}\}_{i=1}^N, \delta_n^1, \{Z_k^{1(j)|2}\}_{i=1}^N, \delta_n^2) + W_k, \end{aligned} \tag{7}$$

where M^{ji} represents the rotation matrix from the location of sensor j to the location of sensor i, T^{ji} represents the translation vector from sensor j to sensor i, and

$$R = COV[W_k] = \begin{bmatrix} P_{k|k}^{1|1} + M^{21} P_{k|k}^{1(1)|2} M^{21^T} & 0 & \dots & 0 \\ 0 & P_{k|k}^{2|1} + M^{21} P_{k|k}^{1(2)|2} M^{21^T} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{k|k}^{N|1} + M^{21} P_{k|k}^{1(N)|2} M^{21^T} \end{bmatrix}. \quad (8)$$

Let

$$G_k^i = \left. \frac{\partial g(Z_{k|k}^i, \delta^i)}{\partial \delta^i} \right|_{\delta^i = \delta_n^i}, \quad (9)$$

where

$$\frac{\partial g(Z_k^i, \delta^i)}{\partial \delta^i} = \begin{bmatrix} -\cos(\theta_k^i - \delta_\theta^i) & (r_k^i - \delta^i) \sin(\theta_k^i - \delta_\theta^i) \\ \dot{\theta}_k^i \sin(\theta_k^i - \delta_\theta^i) & r_k^i \dot{r}_k^i \sin(\theta_k^i - \delta_\theta^i) + (r_k^i - \delta_r^i) \cos(\theta_k^i - \delta_\theta^i) \\ -\sin(\theta_k^i - \delta_\theta^i) & -(r_k^i - \delta^i) \cos(\theta_k^i - \delta_\theta^i) \\ \dot{\theta}_k^i \cos(\theta_k^i - \delta_\theta^i) & -r_k^i \dot{r}_k^i \cos(\theta_k^i - \delta_\theta^i) + (r_k^i - \delta_r^i) \sin(\theta_k^i - \delta_\theta^i) \end{bmatrix}. \quad (10)$$

The numerical iteration for the nonlinear least squares estimate is given by

$$\begin{aligned} B_{n+1} &= B_n + R_{B_n} \bar{G}_k^T R^{-1} [Y_k - \bar{g}(\{Z_k^{j|1}\}_{i=1}^N, \delta_n^1, \{Z_k^{1(j)|2}\}_{i=1}^N, \delta_n^2)] \\ &= B_n - R_{B_n} \bar{G}_k^T R^{-1} [\bar{g}(\{Z_k^{j|1}\}_{i=1}^N, \delta_n^1, \{Z_k^{1(j)|2}\}_{i=1}^N, \delta_n^2)], \end{aligned} \quad (11)$$

and

$$Z_k^{j|i} = h(X_{k|k}^{j|i}), \quad (12)$$

$$\bar{G}_k = \begin{bmatrix} G_k^{1|1} & -G_k^{1(1)|2} \\ G_k^{2|1} & -G_k^{1(2)|2} \\ \vdots & \\ G_k^{N|1} & -G_k^{1(N)|2} \end{bmatrix}, \quad (13)$$

$$R_{B_n} = (\bar{G}_k^T R^{-1} \bar{G}_k)^{-1}. \quad (14)$$

As the gradient descent selects values for B_n, $\bar{g}(\{Z_k^{i(j)|1}\}_{i=1}, \delta_{n-1}^1, \{Z_k^{i(j)|1}\}_{i=2}, \delta_{n-1}^2)$ approaches the ideal value of 0. With this, the biases are able to be iteratively solved at the update of each time k. In the case of two sensors, four biases need to be estimated, and at least two tracks on two common targets are required be obtain a reliable estimate of the biases.

With (11), an estimate of the sensor biases can be computed for each frame. However, this value is highly influenced by the measurement variance that impact the reported tracks. To further reduce the error in the bias estimates, the bias estimates from each frame serves as measurements for a Kalman filter with sensor biases as the state. To do this, the resultant bias B and the bias variance R_B are used as measurements for a standard Kalman filter assuming a constant bias value over each iteration.

IV. Track Fusion Methodology

To appropriately identify track pairs for estimating the sensor biases, these track pairs need to be correlated and fused for a single track number on the targets throughout the scenario. With multiple tracks that can correlate, some form of assignment algorithm is necessary. For this simulation, an initial stage of gating track pairs is used to bound the assignment problem. For gating, the track covariances are inflated for the biases and a large chi-square threshold is applied to the Mahalanobis distance between two tracks to ensure that tracks from the same target pass the gating test. The score of the potential correlation of track held by sensor j to the track held by sensor i is given by

$$C_{ij} = \log \frac{1}{\sqrt{2\pi |S_{ij}|}} - (X_k^i - X_k^j)^T S_{ij} (X_k^i - X_k^j), \quad (15)$$

where $S_{ij} = P_i + P_j$ with P_i and P_j denoting the covariances for tracks from sensors i and j, respectively. This cost function is filled out into a 2D cost matrix, and K-best hypotheses are found using Murty's K-best 2D assignment algorithm. For each of these hypotheses, the sensor biases are estimated and applied to the individual hypothesis, and the bias-correct hypotheses are reordered to find the new best hypothesis. Once the potentially correlated sensor tracks from the best hypothesis are corrected for biases, the source tracks are fused into a single system track state according to

$$X_{f,2} = P_{f,N} \sum_{i=1}^{2} P_i^{-1} x_i, \quad (16)$$

$$P_{f,2} = \left(\sum_{i=1}^{2} P_i^{-1} \right)^{-1}. \quad (17)$$

V. Simulation

A simple scenario was devised as the basis of the simulation study. Four tracks spread across four quadrants are seen by two sensors so that the biases are observable. Each of the targets move at a constant speed of 250 m/s from their start position as indicated in Figure 1 towards their final position in the scenario. This configuration enables robust estimation of the sensor biases.

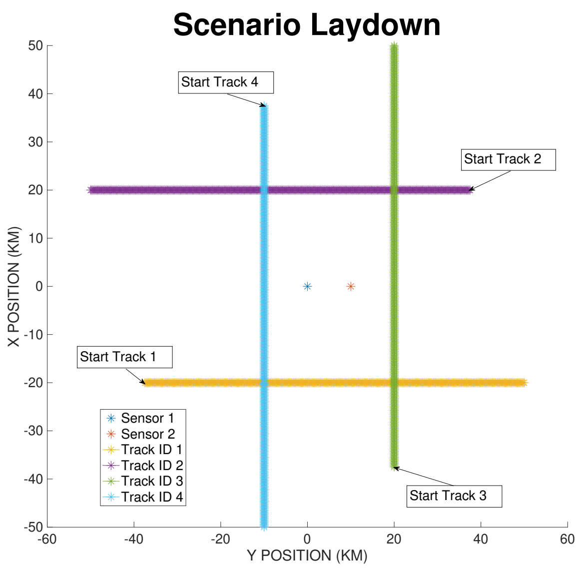

Fig. 1. Laydown of the Scenario

A set of eight different bias configurations as shown in Table I were run. For both sensors in the scenario, the standard deviations of the sensor errors are 5 m in range and 1 mrad in azimuth. The update rate of the sensor trackers 1 s. For the track correlation, the chi-square threshold for track gating was set to 18.4 for a probability of gating of 0.999. The K for the assignment algorithm is 3, since the association is not challenging for this scenario. In complex scenarios, K will be on the order of 200 or more. The process noise standard deviation for the Kalman filter for tracking bias estimates was 0.01 m for the range and 0.1 mrad for the azimuth. The Monte Carlo simulations included 100 runs. Root mean squared error (RMSE) in the bias estimates and normalized estimation error squared (NEES) of the bias estimates were compiled for each Monte Carlo simulation.

TABLE I
BIAS CONFIGURATION TABLE

Scenario	Runs	δ_r^1 (m)	δ_θ^1 (rad)	δ_r^2 (m)	δ_θ^2 (rad)
1	100	0	0	0	0
2	100	25	0.005	0	0
3	100	0	0	25	0.005
4	100	25	0.005	25	0.005
5	100	-25	-0.005	25	0.005
6	100	-25	-0.005	-25	-0.005
7	100	-20	0.001	10	-0.003
8	100	10	0.007	-50	-0.004

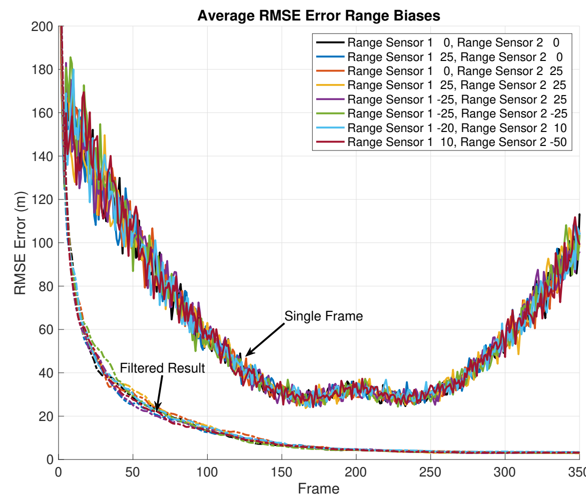

Fig. 2. Root Mean Square Error of Estimated Range Bias

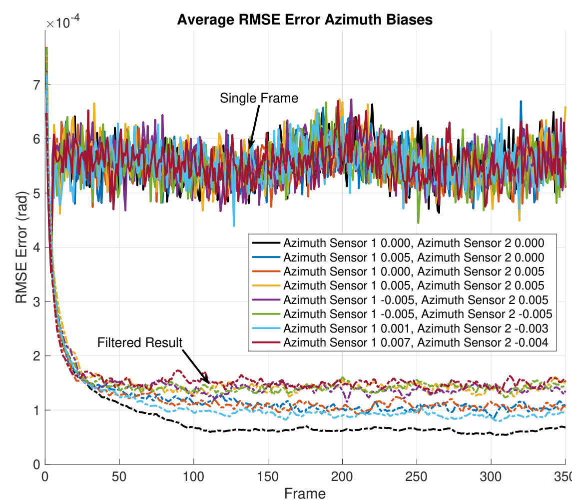

Fig. 3. Root Mean Square Error of Estimated Azimuth Bias

Figures 2 and 3 show the accuracy of individual bias estimates denoted by the solid lines and the accuracy of filtered estimates of the Kalman filter denoted by the dashed lines. The colors match for each simulation across Figures 2, 3, and 4. For the range data, the results of the individual cases are impacted by the range of the target. This trend results from the cross range error in the azimuth changing in magnitude with range of the targets. The RMSE values follow a trend of decreasing as the cumulative range of the targets decrease and then increase as the targets move further away from the sensors. When filtered, the errors in the bias estimates are reduced to an accuracy below the noise floor of the sensors. Since the biases drift slowly, bias estimates converge to a near steady-state condition. The steady-state error in range is about 3 m versus the 5 m noise in the range of the sensors.

For the azimuth data, the accuracy of the individual bias estimates is less dependent on the ranges of the targets. As the accuracy of the range measurements is significantly better than the cross range errors due to the azimuth measurements. The azimuth biases are estimated with far greater accuracy. Even still, the performance of the filtered bias estimates are significantly improvement relative to the individual bias estimates, getting down to an accuracy near 0.1 mrad versus the 1 mrad accuracy of the sensors.

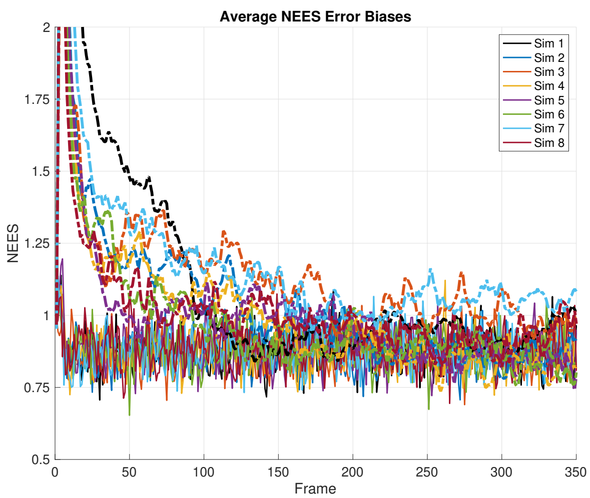

Fig. 4. Normalized Estimation Error of System Tracks to Truth

The NEES of Figure 4 indicate that both the individual bias estimates and filtered estimates achieve the ideal value of 1, since the NEES result has been normalized for the degrees of freedom of the biases. However, it takes time for the filtered results to converge to a value of about 1, which is likely due to the large RMSE values demonstrated over the start of the scenario due to track filter settling. Overall, these results indicate favorable behavior of the algorithm in being able to appropriately estimate biases in the sensor coordinate frame to provide a 360 degree bias that can be applied to every track.

VI. Conclusions

In this paper, a methodology to compute biases in the native, non-linear coordinate frame of a sensor was developed and evaluated. To appropriately identify and correlate tracks, Murty's K-best hypotheses algorithm was used to solve the assignment of tracks from sensor j to sensor i. This assignment was used to compute biases in the polar coordinates of the sensor and apply the biases to the track state for each hypothesis k. The best hypothesis was selected and the resulting system track picture was correlated against the existing system track picture to maintain track continuity across the whole simulation.

Through the results, it is observed that the biases of both sensors are estimated to mitigate their impact on the global track picture. Both the range and azimuth estimates are below the noise floor of the sensors, indicating that these estimates are unlikely to cause negative impact to the tracking results through application of the biases and should improve the overall fused result. Since these biases are slow moving, the update rate the bias estimates could be reduced to mitigate the impact of noisy or inaccurate estimates degrading the solution.

An important part of this method of bias mitigation is having a sufficient number of tracks with enough diversity to observe the biases. When all of the tracks being used for the estimation are closely spaced or in similar perspective to both sensors, the ability to observe the biases diminishes. Future work into characterizing such limitations as well as increasing the number of sensors needs to be performed to look at the efficacy of this algorithm for N sensors. In addition, the impact of maneuvering targets on the performance of bias estimation must be evaluated.

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